AIAA JOURNAL
Vol. 34, No. 2, February 1996

Technical Notes

TECHNICAL NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Prediction of the Far-Field Jet Noise Using Kirchhoff's Formulation

A. S. Lyrintzis*
Purdue University,
West Lafayette, Indiana 47907-1282
and
R. R. Mankbadi†
NASA Lewis Research Center, Cleveland, Ohio 44135

Introduction

COMPUTATIONAL aeroacoustics (CAA) is concerned with the prediction of the aerodynamic sound source and the transmission of the generated sound, starting from the time-dependent governing equations. The full, time-dependent, compressible Navier–Stokes equations describe these phenomena. Although recent advances in computational fluid dynamics (CFD) and in computer technology have made first-principles CAA plausible, the direct extension of current CFD technology to CAA requires that several technical difficulties in the prediction of both the sound generation and its transmission be addressed (e.g., Refs. 1 and 2).

After the sound source has been predicted, several approaches can be used to describe its propagation. The obvious strategy is to extend the computational domain for the full, nonlinear Navier–Stokes equations far enough to encompass the location where the sound is to be calculated. However, if the objective is to calculate the far-field sound, this direct approach requires a prohibitive amount of computer storage and leads to an unrealistic turnaround time (e.g., Ref. 3). One usually has no choice but to separate the computation into two domains, one describing the nonlinear generation of sound, and the other describing the linear propagation of sound.

There are several alternatives to describing the sound propagation after the source has been identified. The first of these approaches is the acoustic analogy (e.g., Ref. 4). In the acoustic analogy, the governing Navier–Stokes equations are rearranged to be of a wave-type form. The far-field sound pressure is then given in terms of a volume integral over the domain containing the sound source. Several modifications to Lighthill's original theory have been proposed to account for the sound–flow interaction or other effects (e.g., Refs. 5 and 6). The major difficulty with the acoustic analogy, however, is that the sound source is not compact in supersonic flows. Errors could be encountered in the calculation of the sound field, unless the computation domain were extended in the downstream direction beyond the location where the sound source has completely decayed (e.g., Ref. 3). Furthermore, an accurate account of the retarded time effect requires that one keep a long record of the time history of the converged solution of the sound source, which again represents a storage problem.

An alternative to the acoustic-analogy approach is the use of the Kirchhoff method, which assumes that the sound transmission is governed by the simple wave equation. The Kirchhoff method consists of the calculation of the nonlinear near- and mid-field, usually numerically, with the far-field solutions found from a linear Kirchhoff formulation evaluated on a surface S surrounding the nonlinear field. The surface S is assumed to include all the nonlinear-flow effects and noise sources. The Kirchhoff method has been used to study various aeroacoustic problems, such as propeller noise, blade–vortex interactions, and so forth. A recent review of the uses of the Kirchhoff method is given by Lyrintzis.[7] The sound pressure and its normal and time derivatives are assumed to be given over the surface enclosing the nonlinear region wherein the sound source is generated. The sound pressure can then be obtained in terms of a surface integral of the surface pressure, which is usually evaluated when CFD techniques are used. This approach seems to have the potential to overcome some of the difficulties associated with the acoustic-analogy approach and is investigated here. Other investigators[8,9] have examined the application of the Kirchhoff method in jet aeroacoustics for a stationary Kirchhoff surface. Here we extend the method to a subsonically moving surface and employ the frequency-domain formulation. In addition we examine several issues related to the accuracy of the calculations. The reader is referred to Ref. 10, which is a longer version of the current document.

Kirchhoff Formulation

A Kirchhoff surface is assumed to enclose all the nonlinear effects and sound sources. Outside this surface the flow is linear and is governed by the convective wave equation

$$\nabla^2 p - \frac{1}{c_\infty^2}\left(\frac{\partial}{\partial t} + U_\infty \frac{\partial}{\partial x}\right)^2 p = 0 \tag{1}$$

If the background velocity U_∞ is zero, the above equation reduces to a simple wave equation. In the classical Kirchhoff formulation for stationary ambient fluid, the far-field sound pressure is given by a surface integral involving the pressure and its normal derivative at the surface. Using the Green's-function method, Morino et al.[11,12] extended this approach to account for moving surfaces. Farassat and Myers[13] derived an alternative Kirchhoff formula using generalized derivatives to evaluate the solution around a subsonically moving surface. The formulations are equivalent.

The pressure field can be expressed by the use of surface integrals [see Ref. 12, Eq. (3)]:

$$p(x, y, z, t) = -\frac{1}{4\pi}\int_{S_o}\left[\frac{p}{r_o^2}\frac{\partial r_o}{\partial n_o} + \frac{1}{r_o}\frac{\partial p}{\partial n_o} + \frac{1}{c_\infty r_o \beta^2}\frac{\partial p}{\partial t}\left(\frac{\partial r_o}{\partial n_o} - M_\infty \frac{\partial x_o'}{\partial n_o}\right)\right]_\tau \mathrm{d}S_o \tag{2}$$

where

$$r_o = \{(x-x)^2 + \beta^2[(y-y')^2 + (z-z')^2]\}^{\frac{1}{2}}$$

$$\tau = \frac{[r_0 - M_\infty(x-x')]}{c_\infty \beta^2}, \qquad \beta = \left(1 - M_\infty^2\right)^{\frac{1}{2}}$$

Received Dec. 2, 1994; presented as Paper 95-0508 at the AIAA 33rd Aerospace Sciences Meeting, Reno, NV, Jan. 9–12, 1995; revision received Oct. 3, 1995; accepted for publication Oct. 6, 1995.

*Associate Professor. Senior Member AIAA.
†Senior Scientist and Leader, Computational Aeroacoustics. Associate Fellow AIAA.

M_∞ is the freestream Mach number, c_∞ is the speed of sound, a prime denotes a point on the Kirchhoff surface, and τ denotes evaluation at the retarded time $t' = t - \tau$. The direction of r_o (needed for the evaluation of the normal derivatives) is from the observer toward the Kirchhoff surface element. The subscript o denotes evaluation at the Prandtl–Glauert transformation directions:

$$x_o = x, \qquad y_o = \beta y, \qquad z_o = \beta z$$

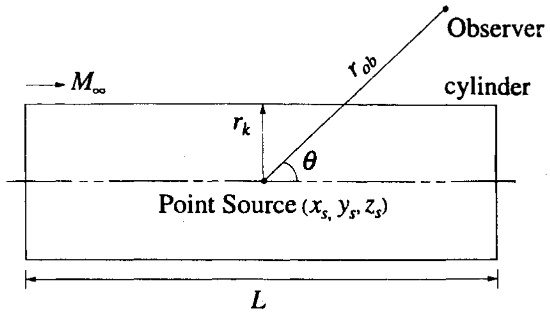

Fig. 1 Kirchhoff surface used to obtain jet-noise calculations.

and $n_o = (n_x, n_y, n_z)$ is the outward normal to the surface S_o. The preceding equation describes the sound pressure at a point (x, y, z) in terms of the information prescribed on the Kirchhoff surface. The numerical simulations can provide the pressure and its normal and time derivatives. It should be noted that the first term has a $1/r^2$ dependence with the distance r, and it is not significant in the far field.

An alternative approach to that above is to perform first a Fourier analysis of the numerically calculated data and to work then with the Kirchhoff formula in the frequency domain. In this approach, we write the surface pressure as $p[(x, y, z), t] = \mathcal{R}\{[\hat{p}(x, y, z)]e^{-i\omega t}\}$. Substituting in Eq. (2) and accounting for the retarded time, we obtain

$$p(x, y, z, t) = \mathcal{R}\Bigg(-\frac{\exp(-i\omega t)}{4\pi} \int_{S_o} \left[\frac{\hat{p}}{r_o^2}\frac{\partial r_o}{\partial n_o} + \frac{1}{r_o}\frac{\partial \hat{p}}{\partial n_o} + \frac{-i\omega}{c_\infty r_o \beta^2}\hat{p}\left(\frac{\partial r_o}{\partial n_o} - M_\infty \frac{\partial x'}{\partial n_o} \right) \right] \times \exp\left\{ i\frac{\omega}{c_\infty \beta^2}[r_o + M_\infty (x' - x)] \right\} \mathrm{d}S_o \Bigg) \tag{3}$$

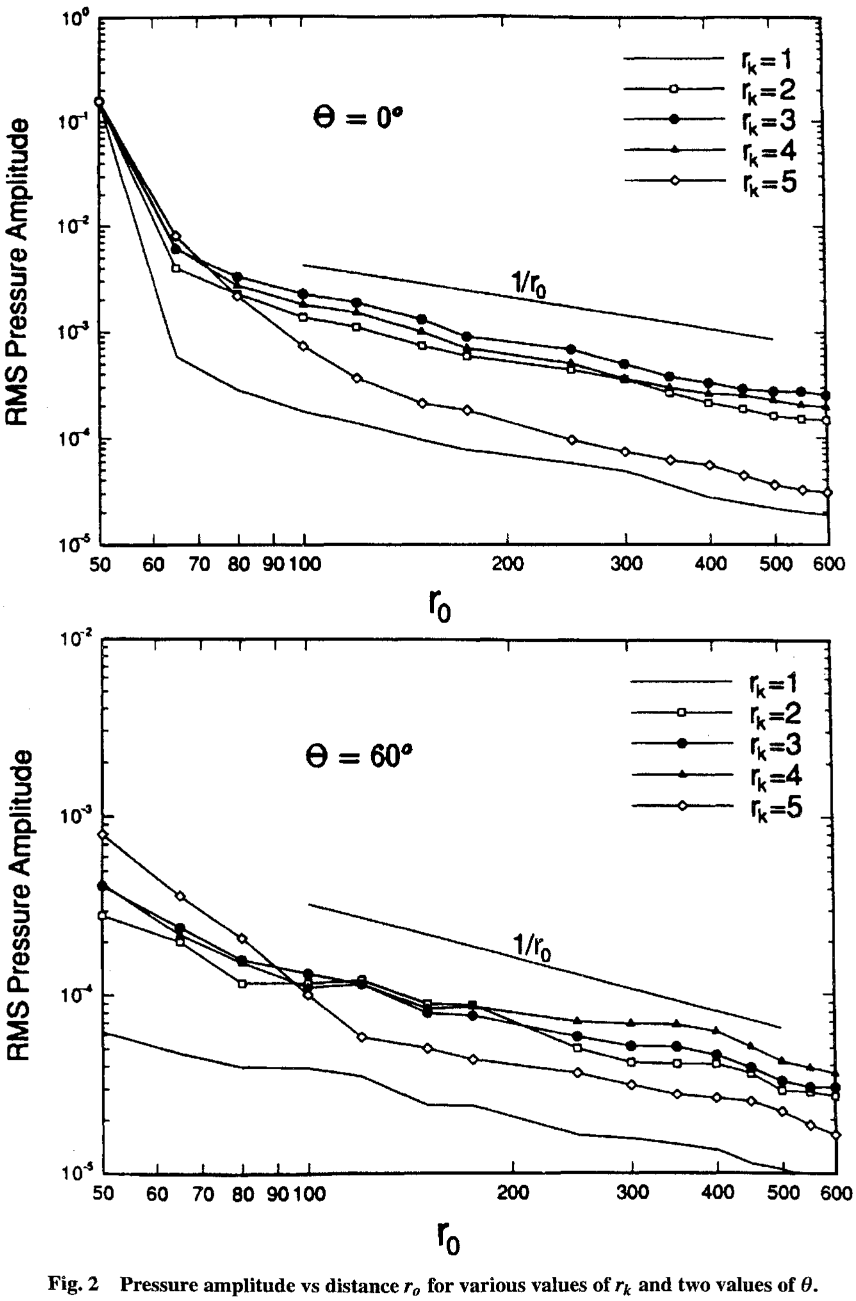

Fig. 2 Pressure amplitude vs distance r_o for various values of r_k and two values of θ.

This is the Kirchhoff formulation for a uniformly moving surface in the frequency domain. Equation (3) is shown here for the first time, to our knowledge, although a similar expression is given by Atassi et al.[14]

Point-Source Test Case

A point-source test case was developed. A point source was placed in the middle of the cylinder at the position (x_S, y_S, z_S), as shown in Fig. 1. Angle θ is also shown in Fig. 1. The sound pressure resulting from a point source was compared by means of values obtained with the Kirchhoff approach and with the exact solution. We took the cylinder length to be $10r_k$ (r_k is the radius of the Kirchhoff surface) and the speed of sound to be 1. The number of data points taken on the cylindrical Kirchhoff surface was 200 points in the x direction and 60 points in the azimuthal direction. For the bases, we took 50 data points in the radial direction; the outer-flow Mach number was 0.8, and frequency was $\omega = 1$ (other frequencies were also tried). As a first check we placed a point inside the Kirchhoff surface, and the result was very close (i.e., $\sim 10^{-4}$) to zero, as expected. We then placed an observer at a distance of $r_{\text{ob}} = 100r_k$. The code for the point source is very fast; it requires only approximately 15 s of CPU time on a Sun SPARC 5 workstation for each observer position for the above data and 50 points per period.

Typical calculations show that the dispersion error is not significant unless a very low number of points per period are used. Time and normal derivatives are evaluated numerically. Fifty points per period are needed for the error to be less than 0.1%, and 18–20 points are needed for the error to be less than 1%. For the normal derivative we found that, if $\mathrm{d}r < 0.1r_k$ (which is not a restrictive condition), the errors are less than 0.1% for 50 points per period. The number of points per wavelength is also an important parameter because it can be used to determine the upper-frequency limit of the Kirchhoff formulation: given a mesh (used both for the CFD and the Kirchhoff formulations), the frequency corresponding to 8–12 points per wavelength can be found, and it should represent the upper limit at which Kirchhoff results are reliable. We found that 12 points per wavelength gave an error of less than 0.5%. Finally, results for the frequency-domain approach were similar to the results from the time-domain approach. Thus the frequency-domain approach is a feasible alternative to that of the time domain.

Results from a Large-Eddy-Simulation Jet Code

We present here some exploratory results for the application of the Kirchhoff method in supersonic-jet aeroacoustics. A Navier–Stokes large eddy simulation (LES) code[3] is used for the computation of the aerodynamic near field. The LES code uses the 2–4 MacCormack method for the solution of the axisymmetric Navier–Stokes equations by the use of LESs. (More details can be found in Ref. 3.) The jet Mach number is 1.5, the exterior Mach number is 0, and the Strouhal number is 0.125. The Navier–Stokes approach with LES uses 256 points per period. The Kirchhoff surface S is a cylinder around the jet; the base surfaces are not included in the calculation. The cylinder length is 50 jet radii along which are 150 points. This mesh is fine enough for CAA calculations. One important issue to be resolved is the determination of the proper positioning of the Kirchhoff surface. Five Kirchhoff surfaces with radii r_k ranging from 1 to 5 jet radii were tried herein. The results are from time-domain calculations. However, frequency-domain calculations yield almost identical results. [The frequency range is $(3\pi/16) - 128(3\pi/16)$.] Both Kirchhoff codes require a very small fraction (i.e., seconds vs hours) of the CPU time required for the CFD calculation.

Figure 2 shows the pressure amplitude vs the distance for various values of r_k at angles of 0 and 60 deg (the angle is measured from the jet axis at the jet exist). A logarithmic scale is used, and the line $1/r$ is also shown. We can observe that all results look reasonable. At large distances all signals drop as $1/r$, as expected. The solution on the Kirchhoff surface should satisfy the linear wave equation. As we move from the nonlinear aerodynamic near field toward the acoustic far field, the signals start to become linear. We can see from Fig. 2 that the signals from a Kirchhoff surface at $r_k = 1$ jet radius are the lowest, because the Kirchhoff surface is in the nonlinear aerodynamic near field. If we place the Kirchhoff surface in the linear region, then the solution is independent of the placement of the Kirchhoff surface. We can also see from Fig. 2 that the signals for $r_k = 2$–4 jet radii give results that are very close, showing that this is approximately the linear region of the CFD calculation. The signals at $r_k = 5$ are lower, because the mesh used is coarser at this distance, and the signals start to become diffused. Thus a fine grid is needed in the CFD solution to ensure proper wave resolution at the beginning of the linear region, so that a Kirchhoff surface could be placed there. Note that some errors are expected at low values of θ, because of the omission of the cylinder-base surface. Calculations with a point source have shown that the effect of the base surfaces near $\theta = 0$ deg is not significant for the pressure amplitude, but it is significant for the calculation of the correct phase. Further work needs to be done on including the contribution of the base surface by the use of modeling techniques.

Concluding Remarks

The Kirchhoff method consists of the calculation of the nonlinear near- and midfields, usually numerically, with the far-field solutions found from a linear Kirchhoff formulation evaluated on a surface S surrounding the nonlinear field. The surface S is assumed to include all the nonlinear-flow effects and noise sources. The separation of the problem into linear and nonlinear regions permits the use of the most appropriate numerical methodology for each. The method is simple and accurate. Both time-domain and frequency-domain formulations are developed here. The method is set up for use in jet aeroacoustics. Some test results from the use of a point source are shown here and several accuracy issues (i.e., the number of points per cycle, number of points per wavelength, and mesh spacing for the normal derivative) are discussed. Furthermore, the accurate prediction of the normal derivative of the normal derivative of the pressure required for the Kirchhoff formulation could lead to inaccuracies.[12] However, for the point-source cases tested here, the accuracy of the normal derivative seems to be adequate. Noted that the solution on the Kirchhoff surface should satisfy the wave equation. For the actual jet-flow case there might be problems if the domain were extended far enough for the solution to satisfy accurately the wave equation without the diffusion that results from coarse grids. The region of 2–4 radii seems to behave linearly for our test case and is the proper region for the placement of the Kirchhoff surface. However, our results do not include the contribution of the cylinder base, and it could be important for observer positions with low angles from the jet axis. In addition, in the area downstream of the cylinder and near the axis, the velocity is not constant, so the convective wave equation is not satisfied; thus additional errors are introduced for observer positions with low values of θ. Further work is needed in the above areas, but our initial results show promise.

Acknowledgments

The first author acknowledges that some of the calculations were performed at the computational facilities of the Minnesota Supercomputer Institute under a resource Grant from Cray, Inc. Most of this work was completed in the summer of 1993, while the first author was visiting the Institute for Computational Mechanics at NASA Lewis Research Center. Finally, the authors acknowledge the help of S. I. Shih from NASA Lewis Research Center and A. Pilon from the University of Minnesota for running some test cases.

References

[1]Hardin, J., and Hussaini, M. Y., *Computational Aeroacoustics*, Springer–Verlag, New York, 1993.

[2]Mankbadi, R. R., *Transition, Turbulence and Noise*, Kluwer Academic, Norwell, MA, 1994, Chap. 9.

[3]Mankbadi, R. R., Hayder, M. E., and Povinelli, L. A., 1994, "The Structure of Supersonic Jet Flow and Its Radiated Sound," *AIAA Journal*, Vol. 32, No. 5, 1994, pp. 897–906.

[4]Lighthill, M. J., "On the Sound Generated Aerodynamically, Part I, General Theory," *Proceedings of the Royal Society of London A*, Vol. 211, 1952, pp. 567–587.

[5]Lilley, G. M., "On the Noise from Jets: Noise Mechanisms," AGARD-CP-131, March 1974, pp. 13.1–13.12.

[6]Phillips, O. M., "On the Generation of Sound by Supersonic Turbulent Shear Layers," *Journal of Fluid Mechanics*, Vol. 9, No. 1, 1960, pp. 1–28.

[7]Lyrintzis, A. S., "The Use of Kirchhoff's Method in Computational Aeroacoustics," *ASME Journal of Fluids Engineering*, Vol. 116, No. 4, 1994, pp. 665–676.

[8]Soh, W. Y., "Unsteady Jet Flow Computation—Towards Noise Prediction," AIAA Paper 94-0138, Jan. 1994.

[9]Mitchell, B. E., Lele, S. K., and Moin, P., "Direct Computation of the Sound Generated by Vortex Pairing in an Axisymmetric Jet," AIAA Paper 95-0508, Jan. 1995.

[10]Lyrintzis, A. S., and Mankbadi, R. R., "On the Prediction of the Far-Field Jet Noise Using Kirchhoff's Method," AIAA Paper 95-050, Jan. 1995.

[11]Morino, L., "Mathematical Foundations of Integral Methods," *Computational Methods in Potential Aerodynamics*, edited by L. Morino, Springer–Verlag, New York, 1985, pp. 271–291.

[12]Morino, L., and Tseng, K., "A General Theory of Unsteady Compressible Potential Flows with Applications to Airplanes and Rotors," *Developments in Boundary Element Methods*, edited by P. K. Banerjee and L. Morino, Vol. 6, Elsevier Applied Science, Barking, England, UK, 1990, pp. 183–245.

[13]Farassat, F., and Myers, M. K., "Extension of Kirchhoff's Formula to Radiation from Moving Surfaces," *Journal of Sound and Vibration*, Vol. 123, No. 3, 1988, pp. 451–460.

[14]Atassi, H. M., Dusey, M., and Davis, C. M., "Acoustic Radiation from a Thin Airfoil in Nonuniform Subsonic Flows," *AIAA Journal*, Vol. 31, No. 1, 1993, pp. 12–19.

Boundary Conditions for the Vorticity–Velocity Formulation of Navier–Stokes Equations

Alexander L. Ni*
Asea Brown Boveri, Ltd., CH-5405 Baden, Switzerland

Introduction

THE no-slip boundary conditions for the vorticity–velocity formulation or, more precisely, the $u - \omega$ integro-differential formulation of the Navier–Stokes (NS) equations are considered in this paper. The status of the problem has been given in the recent reviews by Gresho[1] and Gatski.[2] The present Note is concerned with the no-slip boundary conditions within the framework of the boundary-element method that has been in development for the past two decades by Wu,[3,4,8] Wu and Gulcat,[5] Wu and Thomson,[6] and Wang and Wu.[7] This method has appeared to be rather fruitful. Important problems for both practice and basic research have been solved with this method. Wu et al.[3–8] presented the equation that is sometimes used to represent a solution on NS equations for given time instant in integral form. "Sometimes" is written here because the solver given by those authors does not generally speaking meet the no-slip boundary conditions. Gresho[1] wrote in his review that under some circumstances the equation "... may not be strictly valid, and some mathematical trickery seems to have crept in. Woe is Wu? No, because in practice it is quite properly (and cleverly) utilized in other ways, and the suggested illegitimacy is in fact vindicated." However, the problem for the general case remains unsolved. How to meet the no-slip conditions? This Note approaches these questions by reduction of the problem finally to the system of Fredholm equations of the second kind. The solver of this system gives the distribution of vortices and sources (sinks) on the surface of the body. The integral representation of the solution with the obtained vortices and sources yields the exact instantaneous velocity field that meets the no-slip conditions.

Received March 28, 1995; revision received Oct. 5, 1995; accepted for publication Oct. 9, 1995.

*Guest Scientist, CRBT; currently Principal Researcher, Institute of Chemical Physics, 142432 Chernogolovka, Moscow, Russia.

Flow Equations

The terminology and notations from Ref. 1 are used throughout this paper. The governing equations for an incompressible viscous medium are

$$\nabla \boldsymbol{u} = 0, \qquad \frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\nabla)\boldsymbol{u} = \eta \Delta \boldsymbol{u} - \nabla P \tag{1}$$

The most general initial-boundary-value problem (IBVP) is as follows:[1] Find the velocity $\boldsymbol{u}(x, t)$ and the kinematic pressure $P(x, t)$, i.e., the pressure divided by the density in a bounded domain Ω with a boundary value of $\partial\Omega = \Gamma = \Gamma_D + \Gamma_N$, subject to the boundary conditions for $t > 0$ of

$$\boldsymbol{u} = \boldsymbol{w} \qquad \text{on} \qquad \Gamma_D \tag{2}$$

$$-P + \eta\, \partial u_n/\partial n = 0, \qquad \eta\, \partial u_\tau/\partial n = 0 \qquad \text{on} \qquad \Gamma_N \tag{3}$$

where n indicates the normal component and outward normal direction, t indicates the tangential direction, and wherein if (and only if) $\Gamma = \Gamma_D(\Gamma_N)$, for $\Gamma_N = \emptyset$, $\boldsymbol{w}$ must satisfy $\oint_\Gamma \boldsymbol{n}\boldsymbol{w} = 0$ for $t > 0$, and is subject to the initial condition $\boldsymbol{u}(x, 0) = \boldsymbol{u}_0(x)$ in Ω, where $\nabla \boldsymbol{u}_0 = 0$ in Ω, and $\boldsymbol{n}\boldsymbol{u}_0 = \boldsymbol{n}\boldsymbol{w}_0 = \boldsymbol{n}\boldsymbol{w}(x, t)$, ($x \in \Gamma$ and $t = 0$) on Γ_D.

Apply the curl operator $\nabla \times (\cdots)$ to Eqs. (1) and use the fact that both $\boldsymbol{u}$ and ω are division-free to obtain

$$\frac{\partial \omega}{\partial t} + (\boldsymbol{u}\nabla)\omega = (\omega\nabla)\boldsymbol{u} + \eta\Delta\omega \tag{4a}$$

and the velocity vector

$$\omega = \nabla \times \boldsymbol{u} = \operatorname{rot} \boldsymbol{u} \tag{4b}$$

Vector-Potential-Vorticity ($A - \omega$) Formulation

If one now focuses on the no-slip boundary condition Eq. (2) the vector-potential $\boldsymbol{A}$ and the scalar potential φ by means of

$$\boldsymbol{u} = \operatorname{rot} \boldsymbol{A} + \nabla\varphi \tag{5}$$

to arrive at the second in the pair known as the vector-potential-vorticity ($\boldsymbol{A} - \omega$) formulation

$$\Delta \boldsymbol{A} = -\omega, \qquad \Delta\varphi = 0 \tag{6}$$

It is important that one additional requirement (a gauge condition) be imposed on $\boldsymbol{A}$ for the derivation of Eqs. (6):

$$\operatorname{div} \boldsymbol{A} = 0 \tag{7}$$

The no-slip boundary conditions on the surface of the body in the absolute frame are

$$(\nabla\varphi)_n + (\operatorname{rot}\boldsymbol{A})_n - (\boldsymbol{w})_n = 0, \qquad (\nabla\varphi)_t + (\operatorname{rot}\boldsymbol{A})_t - (\boldsymbol{w})_t = 0 \tag{8}$$

The latter equation is really a vector equation on the surface.

The general representations of the solutions for the Poisson equation for the vector potential and the Laplace equation for the scalar potential (6), are[9]

$$\begin{aligned} \boldsymbol{A}(x, t) &= \boldsymbol{\Phi}(x) + \int_\Gamma \boldsymbol{\mu}(x') F(s)\, \mathrm{d}\Gamma(x') \\ \varphi(x, t) &= \int_\Gamma \nu(x') F(s)\, \mathrm{d}\Gamma(x') \end{aligned} \tag{9}$$

where the vector $\boldsymbol{\Phi}(x) = -\beta_d \int_\Omega \omega(x') F(s)\, \mathrm{d}\Omega(x')$.

The functions $\boldsymbol{\mu}(x')$ and $\nu(x')$, entered in the surface integrals, denote the surface vorticity and the surface sources (sinks), respectively. They should be determined from the boundary conditions. Also here, $F(s)$ is the fundamental solution of the Laplace